Lesson Research Proposal for making connections between algebra and graphing

Date of lesson:31/01/2019
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1. **Title of the Lesson:** Fortnightly Maths

2. **Brief description of the lesson**

In a computer game context, students will have to try to identify intersecting points between a line and a curve using a variety of methodologies.

3. **Research Theme**

Loreto Community School's SSE priorities include a multitude of goals
Teaching, Learning and Assessment

The aims of our Mathematics Department include:

- The promotion of editing in light of personal reflection.
- The development of a collaborative culture through peer observation, impacting positively on teaching, learning and assessment, for both teachers and students alike.

Overall, engaging with lesson study will provide students with an opportunity to engage with mathematics in both a collaborative and reflective nature. Students will be expected to work together to devise a problem with a multitude of solutions. Regardless of the level of sophistication of the solutions found, students can then critique their chosen methodologies based on other methods found. This should provoke a reflective stance among students on prior knowledge and newly acquired information. Participating in lesson study provides our Maths Department with meaningful professional development as we will collaborate together to share ideas and methodologies internally and externally by virtue of other teachers from other schools.

4. **Background & Rationale**

Topic: Connecting Functions to other topics of the syllabus.
Level: Leaving Certificate Ordinary Level.

- We felt that when teaching functions misconceptions among students often arises.
- Relationships between algebra, coordinate geometry, calculus and functions are rarely identified. It is important that students recognise connections between the various topics for improved conceptual understand.
- Many students often struggle with notation that is not typical of functions notation, for example f(x) is more familiar to most, yet if d(t) is provided in a question, difficulties often occur. In

addition, students do not recognise the connection between equations and graphing.

- Our findings above resonate with that of the 2015 Chief Inspector's report of Leaving Certificate Mathematics where it was found that 'At Ordinary level, it was evident that a majority of candidates were unable to formulate and represent information in mathematical form and hence solve problems'.

5. Relationship of the Unit to the Syllabus

Describe how this unit relates to the syllabus/learning outcomes from prior years, for this year and for future learning.

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Secondary School Mathematics *Junior Cert Mathematics* Students should be able to: **Strand 2.2: Co-ordinate Geometry** − Co-ordinate the plane. − Locate points on the plane using coordinates. **Strand 2.3: Trigonometry** − Apply the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances. − Use trigonometric ratios to solve problems involving angles (integer values) between 0˚ and 90˚. − Solve problems involving surds. − Solve problems involving right angled triangles. − Manipulate measure of angles in both decimal and DMS forms. **Strand 4: Algebra** Students should be able to: − Use tables, diagrams and graphs as tools for representing and	**Leaving Certificate Mathematics** Students should be able to: **Strand 2.2: Co-ordinate Geometry** − Coordinating the plane. − Linear relationships in real-life contexts and representing these relationships in tabular and graphical form. − Equivalence of the slope of the graph and the rate of change of the relationship. − Comparing linear relationships in real-life contexts, paying particular attention to the significance of the start value and the rate of change. − The significance of the point of intersection of two linear relationships. **Strand 2.3: Trigonometry** – Use of the theorem of Pythagoras to solve problems (2D only). – Use trigonometry to calculate the area of a triangle.	Calculus: Continued learning after the introduction of first derivatives. − Show how rates of change can be found by referring to the path of the avatar or the shot fired in the question. − Use the problem to introduce the stationary points of a curve. − Introduce finding the slope and equation of a tangent to curve by creating another fire shot that corresponds to a linear equation that will intersect the path of the avatar at one point only (i.e. the parabola). − Prove that second derivatives will verify the maximum point of a curve by referring to this example. − Introduce increasing and decreasing functions and the associated methods

analysing linear and quadratic relations. − Develop and use their own generalising strategies and ideas and consider those of others. − Present and interpret solutions, explaining and justifying. − Explore graphs of motion. − Make sense of quantitative graphs and draw conclusions from them. − Make connections between the shape of a graph and the story of a phenomenon. − Describe both quantity and change of quantity on a graph. − Evaluate expressions of the form: • $ax + by$ • $a(x + y)$ • $x^2 + bx + c$ • axy where $a, b, c, d, x, y \in Z$ – Add and subtract expressions of the form: • $(ax + by + c) \pm \ldots$ $\pm (dx + ey + f)$ • $(ax^2 + bx + c) \pm \cdots \pm (dx^2 + ex + f)$ $where\ a, b, c, d, e, f \in Z.$ **Strand 5: Functions** − Engage with the concept of a function, domain, co-domain and range. − Make use of function notation $f(x) = f : x \rightarrow$, and $y =$ − Interpret simple graphs. − Plot points and lines. − Draw graphs of the following functions and interpret equations of the form f(x) = g(x) as a comparison of functions: • f(x) = $ax + b$, where a, b ∈ Z • f(x) = $ax^2 + bx + c$, where a ∈ N; b, c ∈ Z; x ∈ R • f (x) = $ax^2 + bx + c$, where a, b, c ∈ Z, x ∈ R • f (x) = a^2x and f (x) = a^3x , where $a \in N,\ x \in R$ -Use graphical methods to find approximate solutions where f(x) =	– Solve problems using the sine and cosine rules (2D). – Define sin θ and cos θ for all values of θ. – Define tan θ. – Solve problems involving the area of a sector of a circle and the length of an arc. – Work with trigonometric ratios in surd form. **Strand 4: Algebra** – Evaluate expressions given the value of the variables. – Add and subtract expressions of the form • $(ax + by + c) \pm \cdots \pm (dx + ey + f)$ • $(ax^2 + bx + c) \pm \cdots \pm (dx^2 + ex + f)$ $where\ a, b, c, d, e, f \in Z.$ – Select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • $f(x) = g(x), with\ f(x) = ax + b, g(x) = cx + d\ where\ a, b, c, d \in Q.$ • $f(x) = k\ with\ f(x) = ax^2 + bx + c$ (and not necessarily factorable) where a, b, c ∈ Q and interpret the results. **Strand 5.2: Calculus** − Find derivatives of linear and quadratic functions by rule. – Associate derivatives with slopes and tangent lines. – Apply differentiation to: • Rates of change • Maxima and minima • Curve sketching.	using either the parabola or line or both. − Revision of Co-ordinate Geometry of the Line; slope, midpoint, distance, equation of a line, area of triangle. − Revision of Trigonometry; finding the length of a side, angle measure using trig ratios, sine rule, cosine rule and area of triangle. − Revision of Algebra and Functions simultaneously, showing their connections mathematically and graphically. All revision of topics will demonstrate the interdependent relationships between all topics.

g(x) and interpret the results. − Find maximum and minimum values of quadratic functions from a graph. − Interpret inequalities of the form $f(x) \leq g(x)$ as a comparison of functions of the above form; use graphical methods to find approximate solution sets of these inequalities and interpret the results. − Graph solution sets on the number line for linear inequalities in one variable.		

6. Goals of the Unit

- Student confidence should increase if they attempt a problem that is differentiated for success regardless of mathematical competency.
- Students should recognise that the teaching and learning of mathematics is not a linear process but instead strands are linked.
- A novel problem like this should motivate students to succeed as the problem is relevant to their interests/hobbies.
- Teamwork skills should be developed as students will need to collaborate with the peers to solve the problem.
- Improved confidence among students moving forward with Context and Application questions in the State Exams, which are often perceived as being difficult.

7. Unit Plan

How the research lesson fits into the larger unit plan, helping to show the bigger picture of the whole unit and the progression of learning. Clarify where the research lesson will be taught.

Lesson	Brief overview of lessons in unit
1	Finish the graphing functions. Summarise notation, solving for variables algebraically and graphically.
2	Research Live Lesson which outlines the relationship between Algebra, Trigonometry, Functions and Geometry by encouraging the use of a variety of methods for solving the problem. Ceardaíocht incorporates questions about rate of change of the avatar’s path, i.e. the rate of change of a quadratic.
3	Introduce Calculus and its uses. First derivatives of linear and quadratic functions

	are introduced. Relate to previous problem.
4	Continue with the first derivatives and applications.
5	Max and min points can be referred to using this problem, showing method algebraically by referring to graph.

8. Goals of the Research Lesson:

Looking at the goals of the research lesson itself from two perspectives:

a. Mathematical goals (what students will know/understand by the end of the lesson)

- Students will amend any common misconceptions they may have had by learning how to apply the correct notation, making connections between each topic, and discovering the what calculus is actually used for and its application to the real world.
- Students will recognise and understand the clear relationships present between functions and algebra by using these to represent the flight path of the body and the dart; functions and calculus by finding the rate of change of the moving body through the air; coordinate geometry and functions by finding points and plotting both the flight path of the body and the dart on a graph.
- Students will understand and appropriately use the correct notation present in functions and calculus
- Students will recognise and understand the relationship between equations and graphs, be able to view an equation as a function and accurately plot it on an appropriate graph.

b. Key Skills and Statements of Learning (briefly outline how this lesson targets key skills and statements of learning)

Managing Myself: Students will recognise their personal strengths and weaknesses of the mathematical elements implemented into this lesson. They will reflect on their learning during and after this lesson and suggest ways in which they can improve their learning. Following reflection and assessment of their own learning, students will set personal goals and identify what they need to do going forward to achieve these goals.

Staying Well: Students will gain confidence in their understanding of mathematics and be able to contribute to decision-making within their groups when problem-solving.

Being Creative: Students will think through the problem in a creative manner and try out different approaches and evaluate which method works best in solving the problem.

Working with others: Students will work cooperatively during this lesson by taking on a role within their groups and working together towards achieving their collective goals. They will support each other in their learning and contribute to the decisions as part of the group.

<u>*Managing information and thinking*</u>*:* Students will think creatively and critically about the problem and make connections between what they already know, and the information presented to them in this lesson.

Statements of Learning:
The student:
1. Communicates effectively using a variety of means in a range of contexts in English. (L1)
   - Students will be able to communicate throughout the lesson using mathematical terminology and notation.
4. Creates and presents artistic works and appreciates the process and skills involved.
   - Students will draw to a correct scale the given function and interpret the results.
15. Recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
    - Students will understand that maths is not a stand-alone subject and that it can appear in other subject areas.
16. Describes, illustrates, interprets, predicts and explains patterns and relationships.
    - Students will be able to illustrate the problem on graph paper, interpret the results from their graphs and describe the relationship between both functions.
17. Devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.
    - Students will come up with a wide range of solutions to the given problem ranging from the easiest method to the most complex.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	**Teacher Support**	**Assessment**
This column shows the major events and flow of the lesson, including timings and what will go up on the board.	This column shows additional moves, questions, or statements that the teacher may need to make to help students.	This column identifies (a) what the teacher will look for (formative assessment) that indicates it makes sense to continue with the lesson, and (b) what observers should look for to determine whether each segment of the lesson is having the intended effect.

Introduction Welcome students, outlining the idea of lesson study Divide students into groups (mixed ability) Provide students with the necessary equipment and paper (5 mins).	Resources - Geometry Sets - Log Tables - Problem - Paper - Graph Paper - Calculator - Board Magnets - Computer and Data Projector	N/A
Posing the Task (3-5 mins) Avatar A jumps from a bouncer that is built into the ground. This path can be described by the function $h(t) = 5t - t^2$, where h represents height in metres and t represents time in seconds. Avatar B is lying on the edge of a cliff, aiming a nerph gun, directly above the bouncer at a height of 6.75 metres. Avatar B fires a shot which passes through Avatar A's pathway and hits the ground after 6.75 seconds. Find in as many ways as you can, the possible times and corresponding heights at which Avatar B could have hit Avatar A (leave your answers correct to two decimal places).	- Visual image of the problem will be projected using the data projector. -Students will also be provided with their own copy of the problem and the resources listed above.	Teacher will clarify what students are expected to find i.e. the points of intersection between the 'nerf gunshot' and the avatar's path. A short-time will be provided for students to ask questions.
Student Individual Work Students will work for 20 minutes in differentiated groups to try to come up with as many solutions as possible to solve the problem.	The interactive classroom timer will be displayed for 20 minutes in the data projector. The timer can be found at https://www.online-stopwatch.com/classroom-timers/	Question students about other methods to verify answers throughout the 20 minutes set.

1. Estimation (a) Students will simply guess what distances the nerf gunshot and path intersect using the information that was provided in the problem. (b) Substitution. Students will try to guess the common x-values that sub correctly into both functions.	Encourage all estimation attempts but emphasise that estimations may not be accurate. Validations should also be required.	Observers should note the length of time it takes students to achieve a certain level of success using the trial and error approach.
Estimation Let 1cm = 1m. Measure with a ruler the height of the Cliff where Avatar B is standing. Measure from the base of the cliff 6.75cm Draw in a line connecting where Avatar B is standing at 6.75cm to this 6.75cm on the ground. This gives the path of the shot. Estimate the path of Avatar A by continuing the path given in the diagram. Measure both heights and times where the graphs inter-sect.	Acknowledge this valid approach but encourage other solutions, i.e. 'how could this be illustrated?' Continually remind students that the correct units of measurements must be included in their findings, i.e. seconds and metres.	
2. Substitution Using trial and error, students will estimate values of t and then sub them into both functions to see where common values for h arise.	Commend this methodology, question validity and the practical implications that may arise when using a trial and error approach. Are there other methods that could be more efficient?	

Equation of Avatar A's jump is given:

$h(t) = 5t - t^2 \Rightarrow y = 5x - x^2$

Equation of Avatar B's shot not provided. Therefore, we must find the equation of this line.

Two points on this line are given: $(0, 6.75)$, $(6.75, 0)$

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - 6.75}{6.75 - 0} = -1$

Equation of a line: $y - y_1 = m(x - x_1)$

$y - 6.75 = -1(x - 0)$

$y - 6.75 = -x$

$y = -x + 6.75$

Now have two equations:

1) $y = 5x - x^2$
2) $y = -x + 6.75$

$\Rightarrow 5x - x^2 = -x + 6.75$

$x^2 - 6x + 6.75 = 0$

Using trial and error, guess the possible value for x and substitute into the equation above.

$x = 1$: $(1)^2 - 6(1) + 6.75 = 0$

$1 - 6 + 6.75 = 0$

$1.75 \neq 0$

$x = 2$: $(2)^2 - 6(2) + 6.75 = 0$

$4 - 12 + 6.75 = 0$

$-1.25 = 0$

$x = 1.5$: $(1.5)^2 - 6(1.5) + 6.75 = 0$

$2.25 - 9 + 6.75 = 0$

$0 = 0$

Sub $x = 1.5$ into one of the original equations

$y = -(1.5) + 6.75$

$y = 4.25$

One point of intersection is $(1.5, 4.25)$

Using the trial and error method again to find the second point of intersection.

$x = 3$: $(3)^2 - 6(3) + 6.75 = 0$

$9 - 18 + 6.75 = 0$

$-2.25 = 0$

$x = 4$: $(4)^2 - 6(4) + 6.75 = 0$

$16 - 24 + 6.75 = 0$

$-1.25 = 0$

$x = 4.5$: $(4.5)^2 - 6(4.5) + 6.75 = 0$

$20.25 - 27 + 6.75 = 0$

$0 = 0$

Sub $x = 4.5$ into one of the original equations

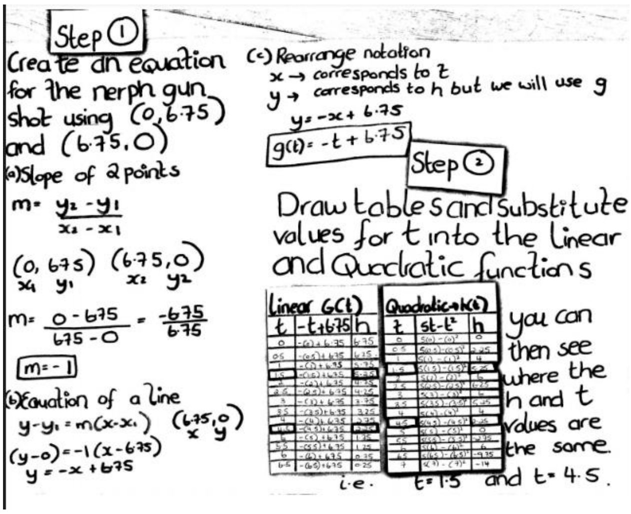
$y = -(4.5) + 6.75$

$= 2.25$

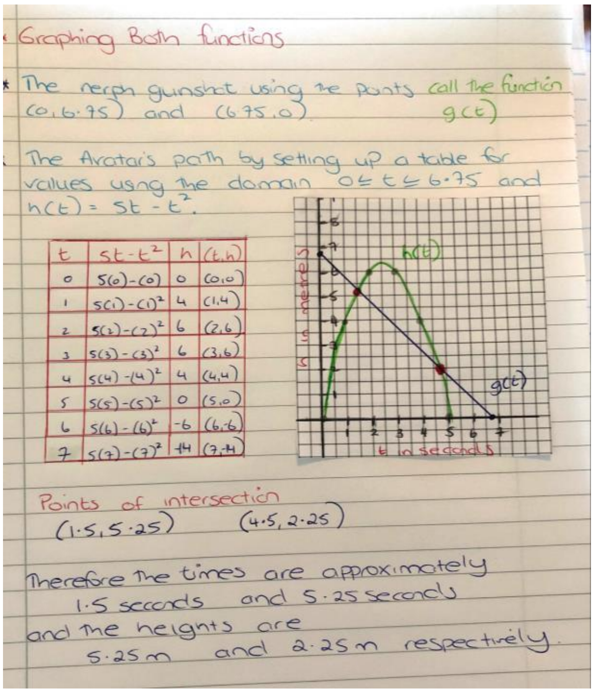
The other point of intersection is $(4.5, 2.25)$

3. Creating a domain for each function and placing the values for t and h into two separate tables

Students will create a function for the nerph gunshot using the two points (6.75, 0) and (0, 6.75) which would correspond to an equation of a line. They would then set up a domain $0 \leq t \leq 6.75$ from the information given in the question, substitute these values into the functions, create two tables and then identify the common values.

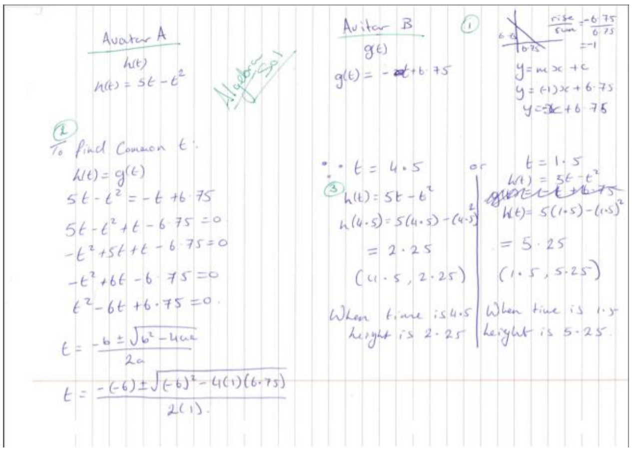


4. Graphing
Students will plot the line and quadratic functions, estimate a domain and find the intersecting points between them.



5. Algebra, Co-ordinate Geometry and Functions

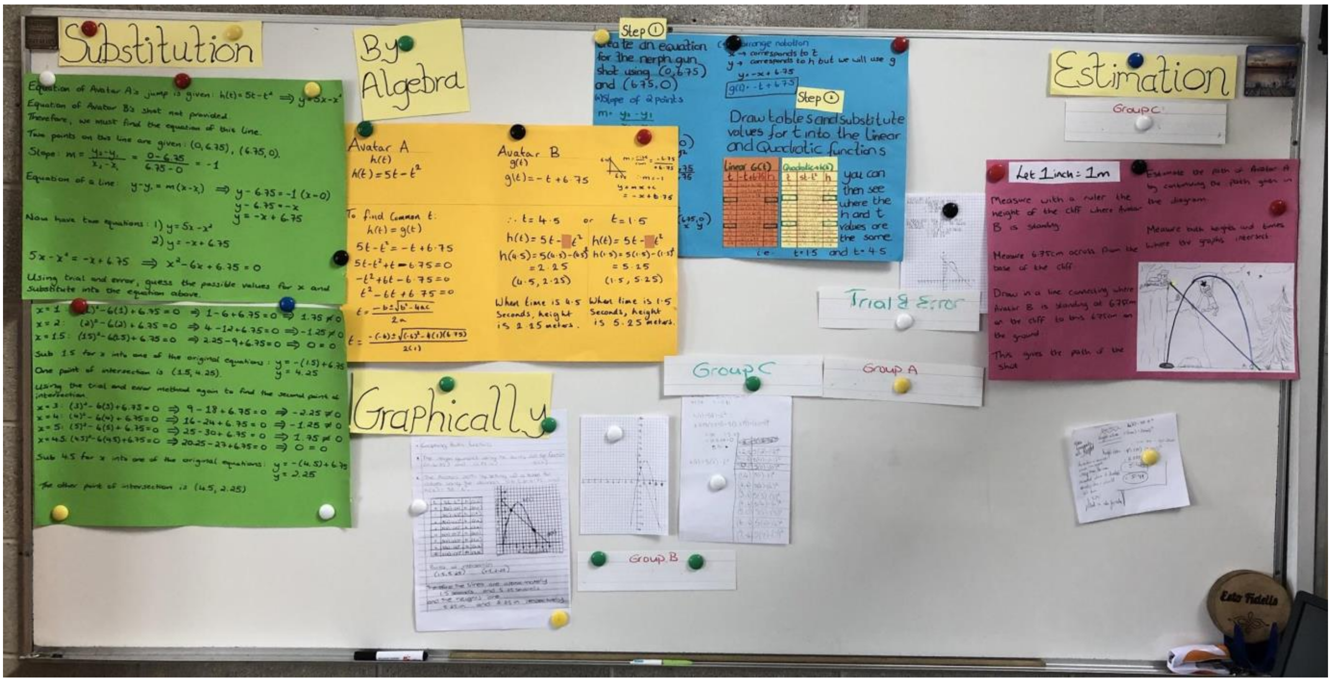
Students will recognise that two intercepts have been provided in the original problem. They then will calculate the slope of the line, using

the formula or Rise/Run. The equation of the line will then be calculated. Students will equate the functions and solve for the unknown co-ordinates i.e. the times and heights.  		
Ceardaíocht /Comparing and Discussing (15 - 20 mins) This section focuses on highlighting how connections between algebra, co-ordinate geometry and functions are made visible in this task. In addition, Ceardaíocht will show the efficiency of each method, particularly emphasising that the algebraic approaches are more valid than estimation/graphing as approximations are often yielded instead of true values.	The various methods will be discussed and compared. Students can be reminded there are possible verification strategies for graphing function by using algebra and vice versa. This question can be used to then introduce Calculus and rates of change by alluding to the distance with respect to time of both the nerph gun shot and the avatar's path. How example, a possible question would be how could you calculate the speed of the nerphs' gun shot? Students could estimate and use distance over time. The teacher will then outline that such rates of change, in the next topic of calculus, can also be found if only limited information is provided.	Students will be expected to justify any of their methods, whether they are correct or incorrect.

Summing up & Reflection (5 mins)	Students will be asked to fill in the reflection sheet provided.	Could students recognise the multi-dimensional aspects to algebra and graphing of functions? Do they see the connections between the various strands of the curriculum? How was the notation difference perceived and did students overcome such obstacles? Did students understand the relevance of the lesson to the forthcoming topic of calculus and rates of change?

10. Board Plan

Board Plan

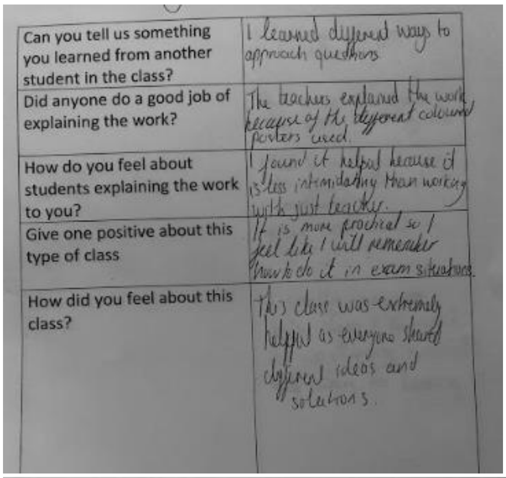


Evaluation and Reflection

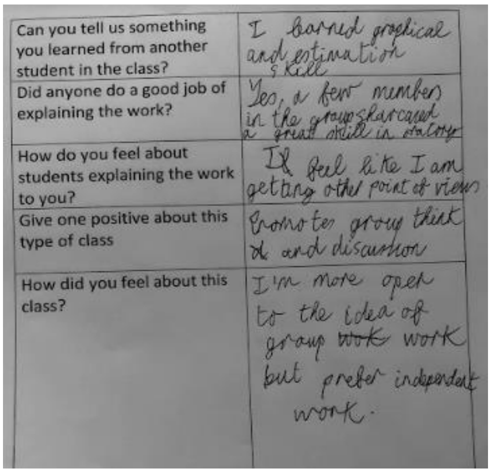
- Overall, the lesson was very successful. One of our main aims was to implement collaborative and reflective practice, not only for students but for ourselves too by engaging in meaningful continued professional development. Designing this lesson made these aims very achievable by attending the meetings and by reviewing/evaluating the problem. Once completed our students then got an opportunity to work with their peers through student-to-student discussions and then by reflecting on their learning.
- Students were every engaged from the offset, yet it appeared that they were apprehensive to work with their peers. It took some time for effective discussion to begin. Most students continually tried to find new methods throughout, however some students gave up halfway into the time allocated for student work.
- Reading the text from the problem caused some problems in the offset. It took some time to focus on what the question was being asked. Some students attempted to differentiate and Distance/Speed/Time methods. Almost all substituted 6.75 into the quadratic function of h(t) but failed to make a logical conclusion about the findings in the offset.
- Students who worked on the image made much more progress than those who did not. Students should be reminded that if an image is not provided with a maths problem, then sketching may aid progress.
- Group work was steered very well as no individual dominated the group throughout, every single student contributed to solving the problem.
- We recognised that the identification of varied function notation can often lead to difficulties.

This lesson certainly helped many students apply a context to the x and y values that arose algebraically and graphically. Initially students did not understand what 'h' and 't' meant in a mathematical level.

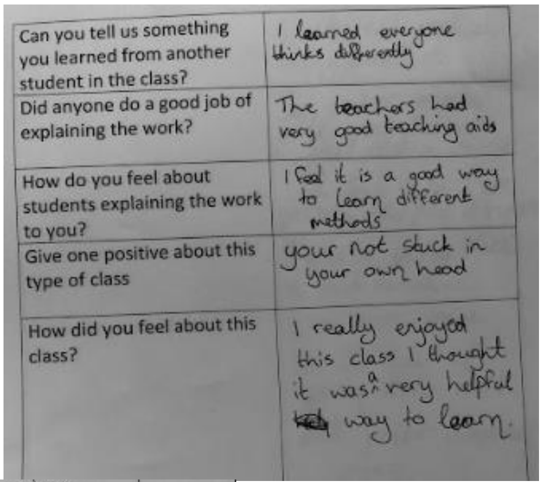
- Students recognised the connection between algebra and graphing during the Ceardaíocht process. At the beginning of the lesson, students were heard saying statements like 'what chapter is this?', and 'I don't know if this is algebra or functions'. However, by the end of the lesson many of the responses on the students' reflection sheets outlined many understood that maths can have multiple solutions.
- During Ceardaíocht, students were apprehensive to come up to the board and were more comfortable discussing their workings from their seats. Furthermore, students were not entirely confident when explaining their findings. It appeared that they found it difficult to put their solution into their own words.
- Below are some examples of our students' reflection sheets:



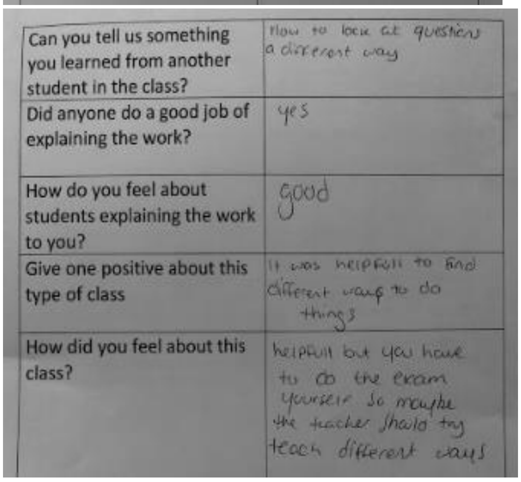
Can you tell us something you learned from another student in the class?	I learned different ways to approach questions
Did anyone do a good job of explaining the work?	The teachers explained the work because of the different coloured posters used.
How do you feel about students explaining the work to you?	I found it helpful because it is less intimidating than working with just teacher.
Give one positive about this type of class	It is more practical so I feel like I will remember how to do it in exam situations.
How did you feel about this class?	This class was extremely helpful as everyone shared different ideas and solutions.



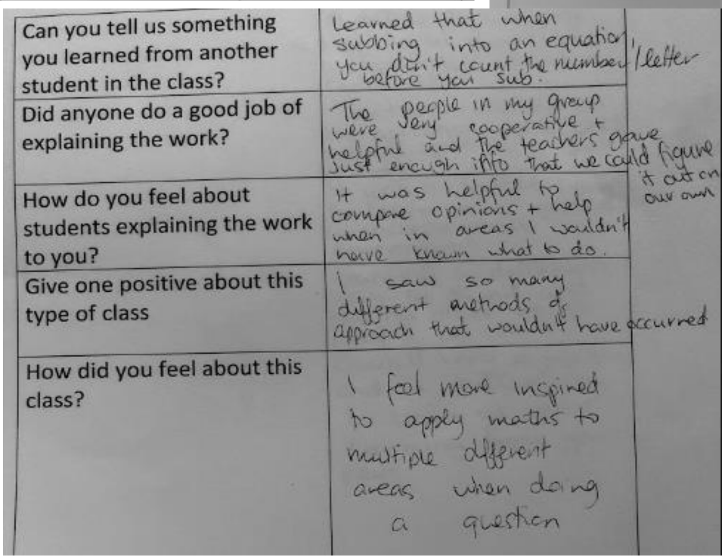
Can you tell us something you learned from another student in the class?	I learned graphical and estimation skill
Did anyone do a good job of explaining the work?	Yes, a few members in the group showcased a great skill in strategy
How do you feel about students explaining the work to you?	I feel like I am getting other point of views
Give one positive about this type of class	Promotes group think and discussion
How did you feel about this class?	I'm more open to the idea of group ~~wok~~ work but prefer independent work.



Can you tell us something you learned from another student in the class?	I learned everyone thinks differently
Did anyone do a good job of explaining the work?	The teachers had very good teaching aids
How do you feel about students explaining the work to you?	I feel it is a good way to learn different methods
Give one positive about this type of class	your not stuck in your own head
How did you feel about this class?	I really enjoyed this class I thought it was a very helpful way to learn.



Can you tell us something you learned from another student in the class?	How to look at questions a different way
Did anyone do a good job of explaining the work?	yes
How do you feel about students explaining the work to you?	good
Give one positive about this type of class	It was helpfull to find different ways to do things
How did you feel about this class?	helpfull but you have to do the exam yourself so maybe the teacher should try teach different ways



Can you tell us something you learned from another student in the class?	Learned that when subbing into an equation, you don't count the number/letter before you sub.
Did anyone do a good job of explaining the work?	The people in my group were very cooperative + helpful and the teachers gave just enough info that we could figure it out on our own
How do you feel about students explaining the work to you?	It was helpful to compare opinions + help when in areas I wouldn't have known what to do.
Give one positive about this type of class	I saw so many different methods & approach that wouldn't have occurred
How did you feel about this class?	I feel more inspired to apply maths to multiple different areas when doing a question

- As a Maths Department, we believe that Lesson Study is a highly effective form of continued professional development. We will continue to take part in the initiative next year as it allows for collaborative professional planning and careful consideration of our SSE targets, new policy reforms and increased knowledge and understanding of the curricula.